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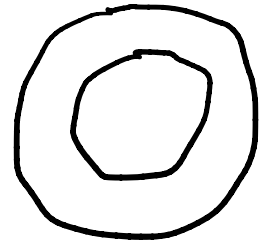
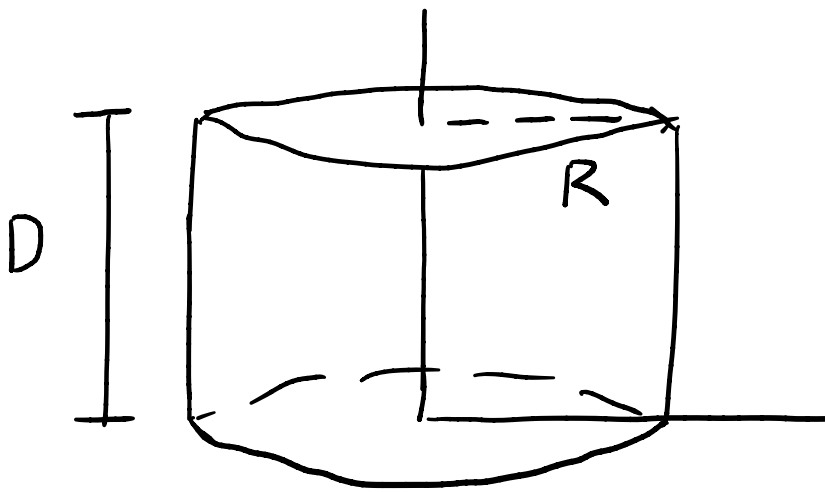
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At point r , amount of chemical is $\frac{k}{1+r^2}$ g/m³ $\times \pi r^2 D$ m³.

$$\Rightarrow y = \frac{\pi k D r}{1+r^2} \text{ g/m} \times r \text{ m}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi k D r_i}{1+r_i^2} \Delta r$$

$$= \pi k D \int_0^R \frac{r}{1+r^2} dr$$

$$= \frac{\pi k D}{2} \ln(1+R^2) \text{ grams}$$

$$\text{Let } u = 1+r^2. \quad du = 2r dr.$$

$$\int_0^R \frac{r}{1+r^2} dr$$

$$= \int_0^R \frac{1}{2u} du$$

$$= \frac{1}{2} \ln|u| \Big|_0^R$$

$$= \frac{1}{2} (\ln|1+R^2| - \ln|1+0|)$$

$$= \frac{\ln(1+R^2)}{2}$$